Indirect Identification of Linear Stochastic Systems with Known Feedback Dynamics

Jen-Kuang Huang*
Old Dominion University, Norfolk, Virginia 23529-0247
Min-Hung Hsiao[†]
Taipei Institute of Technology, Taipei, Taiwan, Republic of China
and
David E. Cox[‡]
NASA Langley Research Center, Hampton, Virginia 23681-0001

An algorithm is presented for identifying a state–space model of linear stochastic systems operating under known feedback controller. In this algorithm, only the reference input and output of closed-loop data are required. No feedback signal needs to be recorded. The overall closed-loop system dynamics is first identified. Then a recursive formulation is derived to compute the open-loop plant dynamics from the identified closed-loop system dynamics and known feedback controller dynamics. The controller can be a dynamic or constant-gain full-state feedback controller. Numerical simulations and test data of a highly unstable large-gap magnetic suspension system are presented to demonstrate the feasibility of this indirect identification method.

Introduction

SYSTEM identification is the process of constructing a mathematical model from input and output data for a dynamic system under testing and characterizing the system behaviors. This technique is important in many disciplines such as economics, communication, system dynamics, and control. In the past few decades, a great variety of system identification methods have been studied extensively.[1-3] The choice of an identification method depends on the nature of the system and the purpose of identification. Most existing system identification methods apply for stable systems without requiring feedback terms for identification purpose. For identifying marginally stable or unstable systems, however, feedback control is required to ensure overall system stability. In many cases, a system, although stable, may be operated in closed loop and it is impossible to remove the existing feedback controller for security or production reasons. In other cases, such as economic and biological systems, the feedback effect may be inherent. Consequently, identification has to be performed on a system operating in closed loop. Previous studies[3] showed that accuracy of identification is not necessarily worse in the presence of feedback; in fact, optimal inputs may very well require feedback terms.

For systems operating in closed loop, there are generally three ways to apply the identification methods.[3,4] One way that can always be applied is to treat the bounded plant input/output data exactly as if they were obtained from an open-loop experiment. This procedure is called direct identification. Another way is to treat the closed-loop system as a whole, and its dynamics can first be identified by some method. Then the open-loop plant dynamics may be determined from the identified closed-loop system dynamics using the knowledge of the feedback controller. This approach is called indirect identification. Further, in some cases, the feedback controller may be considered as part of what is to be identified and then the input and output are considered as a joint process and the output of a system driven by noise only. This approach is called jointly

input–output identification.[5] In this paper, an indirect identification algorithm is presented.

Recently, a method[6] was introduced to identify a state–space model from closed-loop test data by using direct identification. For direct identification, because the input to the plant is partly determined from the feedback, it is difficult to ensure that the input has sufficient frequency richness to excite all of the system's dynamics. It is also found that there is no clear advantage to include the feedback signal in the identification process.[7]

On the other hand, a method[8] was also introduced to identify a state–space model for open-loop system from a finite difference model. The difference model, called the autoregressive with exogeneous input (ARX) model, is derived through Kalman filter theory. Another method[9] is derived to obtain a state–space model from open-loop input/output data using the notion of state observers. This approach can use an ARX model with an order much smaller than that derived through the Kalman filter, but the derivation is based on a deterministic approach. Then projection filters, which were originally derived for deterministic systems,[10] are developed for identification of linear open-loop stochastic systems.[11,12] The relationship between the state–space model and the ARX model is derived based on optimal estimation theory. In this paper, this relationship is derived in a much simpler way through z transform of the ARX model and is applied for closed-loop stochastic systems. The computational efficiency for determining the ARX model depends on the choice of the least-squares algorithms.[2,13,14]

For modeling accuracy, one may use three-stage least-squares algorithm to get improved estimates,[15] but the result will be statistically suboptimal. This suboptimality should become evident in the identification results if the noise-to-signal ratio is increased and interval estimates are computed. In this paper, after obtaining the finite-order ARX model, two recursive formulas are derived. The first one calculates the closed-loop system Markov parameters (pulse response) from the estimated coefficient matrices of the ARX model. For open-loop systems, this recursive form provides the exact solution for deterministic system[2,9] and the optimal solution for stochastic systems.[11,12] The second one computes the open-loop system Markov parameters from the calculated closed-loop Markov parameters and known controller dynamics. For closed-loop systems, these recursive forms derived by using z transform provide the optimal solution instead of the least-squares solution. This is the main contribution of this paper. The proposed indirect identification algorithm can be applied for any dynamic or constant-gain feedback controller. The method is also derived in the stochastic

*Professor, Department of Mechanical Engineering. Member AIAA.
[†]Associate Professor, Department of Mechanical Engineering.
[‡]Research Engineer, Guidance and Control Branch, Flight Dynamics and Control Division.

framework, taking into account the effects of process noise as well as measurement noise.

A similar indirect identification approach was also presented in Refs. 16 and 17. However, they applied to deterministic systems only, and thus the optimal Kalman filter gain used for an estimator could not be identified. Furthermore, in Ref. 16, no recursive form was derived for computing open-loop system dynamics. In Ref. 17, the approach is based on system pulse response. In this paper, a recursive form for computing the open-loop system and Kalman filter Markov parameters is derived for stochastic systems with random excitation. The method can also estimate the Kalman filter gain directly without estimating noise covariances. Like other direct approaches,[18] it is simple in theory, has fewer parameters to estimate, and can prevent the problem of nonuniqueness in estimating process noise covariance.

In the following sections, an indirect identification algorithm is first presented for systems with dynamic feedback controller. Then a special case with constant-gain full-state feedback controller is studied. A simpler identification procedure is described. A matrix is also mentioned to transform the identified state–space model from any arbitrary coordinate to the physical coordinate so that the identified system parameters can be compared to the analytical one. Finally, an example of identifying an unstable large-gap magnetic suspension system is provided with numerical simulations and experimental test data to illustrate the proposed indirect identification method.

Closed-Loop State–Space and ARX Models Relationship

In this section, the relation between a closed-loop state–space and an ARX model is derived by using z transforms. A finite-dimensional, linear, discrete-time, time-invariant system can be modeled as

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $x \in R^{n \times 1}, u \in R^{s \times 1}, y \in R^{m \times 1}$ are state, input, and output vectors, respectively; w_k is the process noise and v_k is the measurement noise; and $[A, B, C]$ are the state–space parameters. Sequences w_k and v_k are assumed Gaussian, white, zero mean, and stationary with covariance matrices Q and R, respectively. One can derive a steady-state filter innovation model[14]:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + AK\varepsilon_k \tag{3}$$

$$y_k = C\hat{x}_k + \varepsilon_k \tag{4}$$

where $\hat{x}_k$ is the a priori estimated state, K is the steady-state Kalman filter gain, and ε_k is the residual after filtering: $\varepsilon_k = y_k - C\hat{x}_k$. The existence of K is guaranteed if the system is detectable and $(A, Q^{1/2})$ is stabilizable.[19]

On the other hand, any kind of dynamic output feedback controller can be modeled as

$$p_{k+1} = A_d p_k + B_d y_k \tag{5}$$

$$u_k = C_d p_k + D_d y_k + r_k \tag{6}$$

where A_d, B_d, C_d, and D_d are the system matrices of the dynamic output feedback controller, p_k is the controller state, and r_k is the reference input to the closed-loop system. Combining Eqs. (3–6), the augmented closed-loop system dynamics becomes

$$\eta_{k+1} = A_c\eta_k + B_c r_k + A_c K_c \varepsilon_k \tag{7}$$

$$y_k = C_c\eta_k + \varepsilon_k \tag{8}$$

where

$$\eta_k = \begin{bmatrix} \hat{x}_k \\ p_k \end{bmatrix}, \quad A_c = \begin{bmatrix} A + BD_dC & BC_d \\ B_dC & A_d \end{bmatrix}, \quad B_c = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$A_c K_c = \begin{bmatrix} AK + BD_d \\ B_d \end{bmatrix}, \quad C_c = [C \quad 0] \tag{9}$$

K_c can be considered as the Kalman filter gain for the closed-loop system and the existence of the steady-state K_c is guaranteed when the closed-loop system matrix A_c is nonsingular. Substituting Eq. (8) into Eq. (7) yields

$$\eta_{k+1} = \bar{A}\eta_k + B_c r_k + A_c K_c y_k \tag{10}$$

where $\bar{A} = A_c - A_c K_c C_c$ and is guaranteed to be asymptotically stable because the steady-state Kalman filter gain K_c exists. The z transform of Eqs. (10) and (8) yields

$$\eta(z) = (z - \bar{A})^{-1}[A_c K_c y(z) + B_c r(z)] \tag{11}$$

$$y(z) = C_c\eta(z) + \varepsilon(z) \tag{12}$$

Substituting Eq. (11) into Eq. (12), one has

$$y(z) = C_c(z - \bar{A})^{-1}[A_c K_c y(z) + B_c r(z)] + \varepsilon(z) \tag{13}$$

The inverse z transform of Eq. (13) with

$$(z - \bar{A})^{-1} = \sum_{i=1}^{\infty} \bar{A}^{i-1} z^{-i}$$

yields

$$y_k = \sum_{i=1}^{\infty} C_c\bar{A}^{i-1} A_c K_c y_{k-i} + \sum_{i=1}^{\infty} C_c\bar{A}^{i-1} B_c r_{k-i} + \varepsilon_k \tag{14}$$

Since $\bar{A}$ is asymptotically stable, $\bar{A}^i \approx 0$ if $i > q$ for a sufficient large number q (discussed in Ref. 12). Thus Eq. (14) becomes

$$y_k \approx \sum_{i=1}^{q} a_i y_{k-i} + \sum_{i=1}^{q} b_i r_{k-i} + \varepsilon_k \tag{15}$$

where

$$a_i = C_c\bar{A}^{i-1} A_c K_c, \qquad b_i = C_c\bar{A}^{i-1} B_c \tag{16}$$

The model described by Eq. (15) is the ARX model, which directly represents the relationship between the input and output of the closed-loop system. The coefficient matrices a_i and b_i can be estimated through least-squares methods from random excitation input r_k and the corresponding output y_k. For a number of data points l, the batch least-square solution is

$$\theta = (\Phi^T \Phi)^{-1}\Phi^T \xi \tag{17}$$

where

$$\Phi = \begin{bmatrix} y_q^T & r_q^T & y_{q-1}^T & r_{q-1}^T & \cdots & y_1^T & r_1^T \\ y_{q+1}^T & r_{q+1}^T & y_q^T & r_q^T & \cdots & y_2^T & r_2^T \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ y_{l-1}^T & r_{l-1}^T & y_{l-2}^T & r_{l-2}^T & \cdots & y_{l-q}^T & r_{l-q}^T \end{bmatrix}$$

$$\xi = [y_{q+1} \quad y_{q+2} \quad \cdots \quad y_l]^T$$

$$\theta = [a_1 \quad b_1 \quad a_2 \quad b_2 \quad \cdots \quad a_q \quad b_q]^T$$

Markov Parameters and State–Space Realization

In this section, the closed-loop system and Kalman filter Markov parameters are first calculated from the estimated coefficient matrices of the ARX model. Then the open-loop system and Kalman filter Markov parameters are derived from the closed-loop system and Kalman filter Markov parameters, and known controller Markov parameters. The open-loop state–space model is realized by using singular-value decomposition for a Hankel matrix formed by the open-loop system Markov parameters. Finally, an open-loop Kalman filter gain is calculated from the realized state–space model and open-loop Kalman filter Markov parameters through least squares.

The z transform of the open-loop state–space model (3) yields

$$\hat{x}(z) = (z - A)^{-1}[Bu(z) + AK\varepsilon(z)] \tag{18}$$

Substituting Eq. (18) to the z transform of Eq. (4), one has

$$y(z) = C(z - A)^{-1}[Bu(z) + AK\varepsilon(z)] + \varepsilon(z)$$

$$= \sum_{k=1}^{\infty} Y(k)z^{-k}u(z) + \sum_{k=0}^{\infty} N(k)z^{-k}\varepsilon(z) \qquad (19)$$

where $Y(k) = CA^{k-1}B$ is the open-loop system Markov parameter; $N(k) = CA^{k-1}AK$, the open-loop Kalman filter Markov parameter; and $N(0) = I$, which is an identity matrix. Similarly, for the dynamic output feedback controller (5) and (6) and the closed-loop state–space model (7) and (8), one can derive

$$u(z) = \sum_{k=0}^{\infty} Y_d(k)z^{-k}y(z) + r(z) \qquad (20)$$

$$y(z) = \sum_{k=1}^{\infty} Y_c(k)z^{-k}r(z) + \sum_{k=0}^{\infty} N_c(k)z^{-k}\varepsilon(z) \qquad (21)$$

where $Y_d(k) = C_d A_d^{k-1} B_d$ is the controller Markov parameter, $Y_c(k) = C_c A_c^{k-1} B_c$ the closed-loop system Markov parameter, and $N_c(k) = C_c A_c^{k-1} A_c K_c$ the closed-loop Kalman filter Markov parameters. Also note that $Y_d(0) = D_d$ and $N_c(0) = I$.

Closed-Loop System and Kalman Filter Markov Parameters

The z transform of the ARX model (15) yields

$$\left(I - \sum_{i=1}^{q} a_i z^{-i} \right) y(z) = \sum_{i=1}^{q} b_i z^{-i} r(z) + \varepsilon(z) \qquad (22)$$

Applying long division to Eq. (22), one has

$$y(z) = \left\{ b_1 z^{-1} + (b_2 + a_1 b_1) z^{-2} + [b_3 + a_1(b_2 + a_1 b_1) \right.$$

$$\left. + a_2 b_1] z^{-3} + \cdots \right\} r(z) + \left\{ I + a_1 z^{-1} + (a_1 a_1 + a_2) z^{-2} \right.$$

$$\left. + [a_1(a_1 a_1 + a_2) + a_2 a_1 + a_3] z^{-3} + \cdots \right\} \varepsilon(z)$$

After comparing with Eq. (21), the closed-loop system and Kalman filter Markov parameters can be recursively calculated from the estimated coefficient matrices of the ARX model:

$$Y_c(k) = b_k + \sum_{i=1}^{k} a_i Y_c(k - i) \qquad (23)$$

$$N_c(k) = \sum_{i=1}^{k} a_i N_c(k - i) \qquad (24)$$

Note that $Y_c(0) = 0$, $N_c(0) = I$, and $a_i = b_i = 0$, when $i > q$. One may obtain Eqs. (23) and (24) from Eq. (16) and the definition of the Markov parameters.[9] However, the derivation is much more complex.

Open-Loop System and Kalman Filter Markov Parameters

Substituting Eq. (20) into Eq. (19) yields

$$y(z) = \left(\sum_{k=1}^{\infty} Y(k)z^{-k} \right) \left(\sum_{k=0}^{\infty} Y_d(k)z^{-k}y(z) \right)$$

$$+ \sum_{k=1}^{\infty} Y(k)z^{-k}r(z) + \sum_{k=0}^{\infty} N(k)z^{-k}\varepsilon(z) = \sum_{k=1}^{\infty} \alpha_k z^{-k}y(z)$$

$$+ \sum_{k=1}^{\infty} Y(k)z^{-k}r(z) + \sum_{k=0}^{\infty} N(k)z^{-k}\varepsilon(z) \qquad (25)$$

where

$$\alpha_k = \sum_{i=1}^{k} Y(i)Y_d(k - i)$$

Rearranging Eq. (25), one has

$$\left(I - \sum_{k=1}^{\infty} \alpha_k z^{-k} \right) y(z) = \sum_{k=1}^{\infty} Y(k)z^{-k}r(z) + \sum_{k=0}^{\infty} N(k)z^{-k}\varepsilon(z) \qquad (26)$$

Similarly, one can apply long division to Eq. (26), and then compare it with Eq. (21), to describe the closed-loop system Markov parameters recursively in terms of the open-loop system and controller Markov parameters $Y_d(k) = C_d A_d^{k-1} B_d$:

$$Y_c(j) = Y(j) + \sum_{k=1}^{j} \alpha_k Y_c(j - k) = Y(j)$$

$$+ \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i)Y_d(k - i)Y_c(j - k) \qquad (27)$$

And the closed-loop Kalman filter Markov parameters can be recursively expressed in terms of the open-loop system and Kalman filter Markov parameters, and controller Markov parameters:

$$N_c(j) = N(j) + \sum_{k=1}^{j} \alpha_k N_c(j - k) = N(j)$$

$$+ \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i)Y_d(k - i)N_c(j - k) \qquad (28)$$

Rearranging Eqs. (27) and (28), one has

$$Y(j) = Y_c(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i)Y_d(k - i)Y_c(j - k) \qquad (29)$$

$$N(j) = N_c(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y(i)Y_d(k - i)N_c(j - k) \qquad (30)$$

Note that $Y_c(0) = 0$ and $N_c(0) = I$. One can easily verify Eqs. (29) and (30) from Eq. (9), and also from the definition of the Markov parameters.

The open-loop state–space model can be realized by using singular-value decomposition for a Hankel matrix formed by the open-loop system Markov parameters.[2,20] Once the open-loop A and C are obtained, one can easily calculate the open-loop Kalman filter gain from the open-loop Kalman filter Markov parameters $N(k) = CA^k K$ in a least-squares sense as follows:

$$K = (O^T O)^{-1} O^T \begin{bmatrix} N(1) \\ \vdots \\ N(k) \end{bmatrix}, \quad \text{where} \quad O = \begin{bmatrix} CA \\ \vdots \\ CA^k \end{bmatrix} \qquad (31)$$

Identification with Output Feedback

In this section, we summarize the procedure of indirect identification algorithm.

1) Estimate the coefficient matrices of the ARX model from closed-loop input/output data by using Eq. (17).

2) Compute the closed-loop system and Kalman filter Markov parameters from the estimated coefficient matrices of the ARX model by using Eqs. (23) and (24), respectively.

3) Compute the open-loop system and Kalman filter Markov parameters from the closed-loop system and Kalman filter Markov parameters, and controller Markov parameters calculated from the known controller dynamics, by using Eqs. (29) and (30), respectively.

4) Realize the open-loop system matrices from the open-loop system Markov parameters by using the singular-value decomposition method.[2,20]

5) Estimate the open-loop Kalman filter gain from the open-loop Kalman filter Markov parameters and the realized system matrices by using Eq. (31).

Identification with Full-State Feedback

In this section, the preceding indirect identification problem is considered for a particular case. If a constant-gain full-state feedback controller is used, the open-loop system can be identified by following a simpler procedure. An open-loop system with a full-state sensor and a constant gain full-state feedback controller can be modeled as

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{32}$$

$$y_k = x_k + v_k \tag{33}$$

$$u_k = -Fy_k + r_k \tag{34}$$

where F is the known constant feedback gain and r_k is the reference input to the closed-loop system. After applying filter innovation model[14] to the open-loop system and eliminating control input u_k, the closed-loop system becomes

$$\hat{x}_{k+1} = (A - BF)\hat{x}_k + Br_k + (AK - BF)\varepsilon_k \tag{35}$$

$$y_k = \hat{x}_k + \varepsilon_k \tag{36}$$

Comparing Eqs. (35) and (36) with Eqs. (7) and (8), one can have $\eta_k = \hat{x}_k$, $A_c = A - BF$, $B_c = B$, $A_c K_c = AK - BF$, and $C_c = I$. Then one can use Eqs. (17), (23), (24), and (31) to identify the closed-loop system matrices and Kalman filter gain. If the identified closed-loop system matrices and Kalman filter gain are described by a quadruplet, $[\hat{A}_c, \hat{B}_c, \hat{C}_c, \hat{A}_c\hat{K}_c]$, one needs to transform it to the same coordinate used in Eqs. (35) and (36), so that the controller dynamics can be removed from the closed-loop system. Since full-state feedback is used, the identified output matrix $\hat{C}_c$ is a square matrix and is generally invertible. Then one may use $\hat{C}_c^{-1}$ as the transformation matrix to transform the identified quadruplet to be $[\hat{C}_c\hat{A}_c\hat{C}_c^{-1}, \hat{C}_c\hat{B}_c, I, \hat{C}_c\hat{A}_c\hat{K}_c]$, where I is an identity matrix. Comparing the transformed quadruplet with Eqs. (37) and (38), one can easily obtain

$$A - BF = \hat{C}_c\hat{A}_c\hat{C}_c^{-1}, \qquad B = \hat{C}_c\hat{B}_c \tag{37}$$
$$AK - BF = \hat{C}_c\hat{A}_c\hat{K}_c$$

The identified open-loop system matrices and Kalman filter gain become

$$A = \hat{C}_c\hat{A}_c\hat{C}_c^{-1} + \hat{C}_c\hat{B}_cF, \qquad B = \hat{C}_c\hat{B}_c$$
$$C = I, \qquad K = A^{-1}(\hat{C}_c\hat{A}_c\hat{K}_c + BF) \tag{38}$$

If sensors are available to provide all of the state information, one can choose a constant-gain controller [e.g., a pole-placement controller or a linear quadratic regulator (LQR)] so that the closed-loop system has the same dimension as the open-loop system. This controller can be designed (e.g., by adjusting the weighting matrices in the LQR controller) so that the closed-loop system is very easy to identify. For example, a closed-loop system with poles located evenly within a desired frequency range with similar damping ratios between 0.4 to 0.7 may be easily identified.

Coordinate Transformation

For any dynamic system, although its system Markov parameter is unique, the realized state–space model is not unique. If one needs to compare the identified state–space model with the analytical model, both models have to be in the same coordinate. In Ref. 7, a unique transformation matrix is derived to transform any realized state–space model to be in a form usually used for a structural dynamic system, so that any identified system parameter can be compared with the corresponding analytical one. This unique transformation matrix exists only when one-half of the states can be measured directly. If this condition is not satisfied, other transformation matrices may exist, but they usually are not unique.

Numerical and Test Example

An example is provided, which consists of numerical simulations and actual hardware tests to validate the feasibility of the proposed closed-loop identification method. The large-angle magnetic suspension test facility (LAMSTF),[21,22] a laboratory-scale research project to demonstrate the magnetic suspension of objects over wide ranges of attitudes, has been developed in NASA Langley Research Center (see Fig. 1). This system represents a scaled model of a planned large-gap magnetic suspension system. The LAMSTF system consists of a planar array of five copper electromagnets, which actively suspend a small cylinder with a permanent magnet core. The cylinder is a rigid body and has six independent degrees of freedom, namely, three displacements (x, y, and z) and three rotations (pitch, yaw, and roll). The roll of the cylinder is uncontrollable and is assumed to be motionless. Five pairs of the light-emitting diodes and light receivers are used to indirectly sense the pitch and yaw angles and the three displacements of the cylinder's centroid. Therefore, the control inputs consist of five currents sent into five electromagnets, and the system outputs are five voltage signals measured from five receivers. The currents in the electromagnets generate a magnetic field, which produces a net force and torque on the suspended cylinder. The motion of the suspended cylinder is, in general, nonlinear. Therefore, only the linear time-invariant perturbed motion about an equilibrium state is considered. Because it is difficult to accurately model the magnetic field and its gradients, the analytical model needs to be improved through identification from experimental data.

The analytical model of the LAMSTF system includes four highly unstable real poles (about 10 Hz) and two low-frequency flexible modes (about 1.27 and 0.16 Hz) (see Table 1). The sampling rate is first chosen to be 10 times the highest frequency (about 10 Hz) of the system to avoid the aliasing problem. However, the identified result shows that the values of the identified system Markov parameters increase too fast because of the unstable poles in the system. The Hankel matrix formed by these limited numbers of system Markov parameters becomes ill conditioned, and an accurate state–space model cannot be realized. Therefore, the sampling rate is increased up to 250 Hz to reduce the increasing speed of the system Markov parameters. The recorded data length is 24 s. The order of the ARX model is chosen (about 13) so that the eigenvalues of the identified model coverage to certain values.[7]

Numerical simulations are performed for a constant-gain full-state feedback controller. The system is stabilized by using a constant-gain full-state feedback LQR designed by using the analytical mode.[22] The open-loop system matrices of the analytical model and the designed weighting matrices Q and R are shown in the Appendix. The standard deviation of the system disturbance and measurement noise is about 1 or 10% of the corresponding

Table 1 Comparison of eigenvalues of continuous-time analytical and identified model

Analytical model	Simulation (1% noise) full-state feedback	Simulation (10% noise) output full-state feedback	Testing output feedback
± 58.78	58.79, −57.77	54.10, −64.93	62.78, −62.07
±57.81	57.76, −56.69	53.80, −39.72	61.23, −60.20
±9.78	9.66, −9.61	−0.59 ± 15.95i	9.84, −16.05
±7.97i	0.34 ± 10.07i	−1.98 ± 12.06i	1.06 ± 8.02i
±0.96i	0.17 ± 1.78i	−2.24 ± 4.14i	0.21 ± 1.64i

Fig. 1 LAMSTF configuration.

a) System Markov parameters

b) Kalman filter Markov parameters
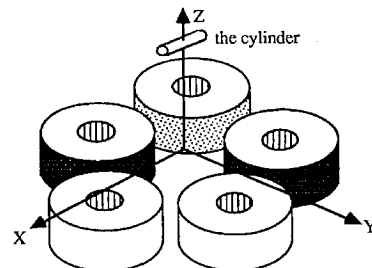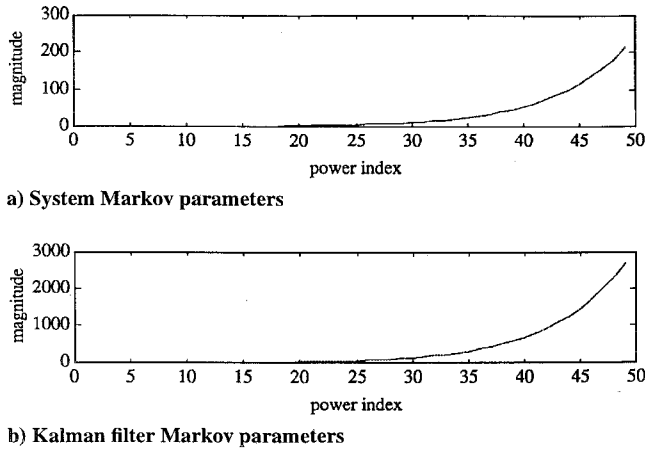
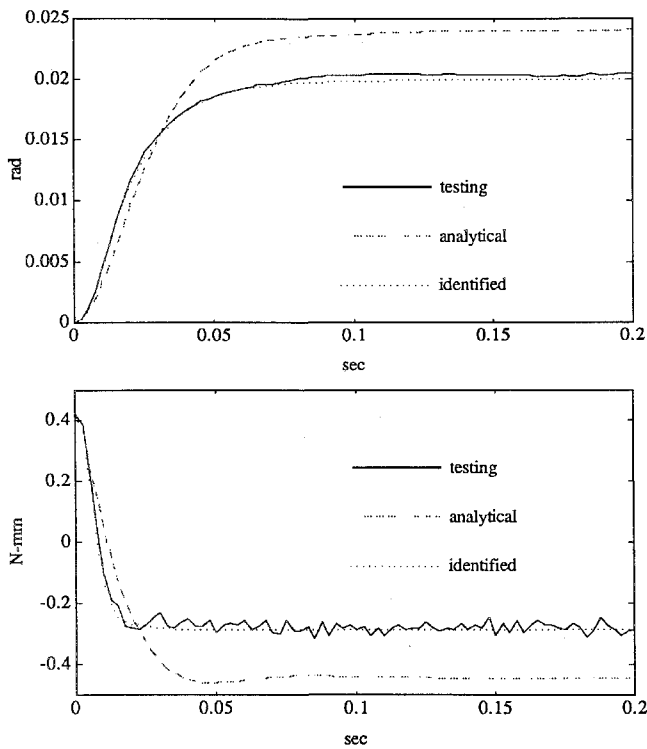Fig. 2 Identified open-loop system and Kalman filter Markov parameters: the (1, 1) element.

Fig. 3 Comparison of pitch step response (top) and control torque (bottom).

input and output signal. The reference inputs contain five uncorrelated random signals. Most of the eigenvalues of the identified model are close to the theoretical values for the 1% noise case (see Table 1). The results also show that the identified state–space model is very close to the analytical one. For the 10% noise case, the first two dominant unstable poles are still fairly close to the theoretical values.

Experiments are also performed for closed-loop identification with a known dynamic output feedback controller. The same constant-gain full-state feedback LQR is used. However, because the rate sensors are not available, the rate information is obtained by calculating the back difference of the sensed position signals. Since the estimation of the rate information is used, the feedback LQR controller becomes a dynamic controller. Therefore, the identification procedure based on dynamic output feedback controller already described is followed. Table 1 compares the eigenvalues of the identified model from experimental data with the analytical model. The system matrices of the identified continuous-time model and the identified discrete-time Kalman gain are shown in

the Appendix. Figure 2 shows the identified open-loop system and Kalman filter Markov parameters.

To evaluate the identified model from experimental data, the simulated step responses with the LQR controller are compared with test data. Figure 3 shows the measured step response and the corresponding control torque in pitch, and computed responses using the analytical model and identified model. The experimental results demonstrate that the experimentally identified model is fairly accurate in predicting the step responses, whereas the analytical model has a deficiency in the pitch axis.

Concluding Remarks

For a stochastic system operating under closed-loop conditions, a method has been introduced to indirectly identify an open-loop state–space model from the closed-loop input/output data without recording the feedback signals. The main contribution is that a recursive form for computing the open-loop system and Kalman filter Markov parameters is derived for stochastic systems with random excitation. This method also estimates the Kalman filter gain directly from the closed-loop input/output data without estimating noise covariances. The identified open-loop state–space model and Kalman filter gain can be used directly for further estimator/controller design.

Acknowledgment

Appendix: Analytical and Identified Model

The analytical model of the large-angle magnetic suspension test facility is

$$\dot{x} = A_m x + B_m u + w \qquad (A1)$$

$$y = C_m x + v \qquad (A2)$$

where

$$x = \left\{ \begin{array}{c} x_p \\ \dot{x}_p \end{array} \right\}, \qquad A_m = \begin{bmatrix} 0_{5 \times 5} & I_{5 \times 5} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0_{5 \times 5} \\ B_2 \end{bmatrix}, \qquad C_m = [\, I_{5 \times 5} \quad 0_{5 \times 5} \,]$$

The state variable x_p includes pitch and yaw angles and three linear displacements of the cylinder's centroid. The matrices A_{21}, A_{22}, and B_2 are

$$A_{21} = \begin{bmatrix} 3341.5 & 0 & -39,392 & 0.0000 & 0.0000 \\ 0 & 3341.5 & -0.0000 & 0.0000 & -0.0000 \\ -9.8070 & -0.0000 & 49.937 & 0.0000 & -0.0251 \\ -0.0000 & 0.0000 & 0.0000 & 95.577 & -0.0000 \\ -0.0000 & -0.0000 & -0.0251 & -0.000 & -0.9132 \end{bmatrix}$$

$$A_{22} = 0_{5 \times 5}$$

$$B_2 = \begin{bmatrix} 38.370 & 38.370 & 38.370 & 38.370 & 38.370 \\ 0 & 89.802 & 55.514 & -55.514 & -89.802 \\ 0.2214 & -0.1527 & 0.0785 & 0.0785 & -0.1527 \\ 0 & 0.1215 & -0.1967 & -0.1967 & -0.1215 \\ -0.2767 & -0.0855 & 0.2239 & 0.2239 & -0.0855 \end{bmatrix}$$

The weighting matrices used for LQR control are

$$Q = \mathrm{diag}[1e3, 1e3, 2e8, 2e8, 2e8, 0.1, 0.1, 2e4, 2e4, 2e4]$$

and

$$R = I_{5 \times 5}$$

The corresponding matrices of the identified system from experimental data are

$$A_{21} = \begin{bmatrix} 3766.6 & 70.325 & -75,117 & 1924.3 & -27,136 \\ -117.61 & 3923.4 & -1077.2 & -29312 & -3690.0 \\ -3.2179 & 5.7394 & 136.28 & -22.192 & 64.823 \\ -0.9768 & 12.871 & -11.237 & -88.650 & 7.9619 \\ 9.8282 & 0.1383 & 51.006 & -7.8415 & -66.471 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} -0.3676 & 0.2909 & -149.27 & -6.4746 & -82.993 \\ 1.8430 & 1.5609 & 176.17 & 9.8511 & 66.933 \\ -0.0034 & 0.0042 & -5.1229 & -0.4162 & -0.5839 \\ 0.0075 & 0.0084 & 0.0980 & -0.1222 & -0.0837 \\ -0.0317 & 0.0083 & -3.7143 & 0.1091 & -2.1338 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 65.913 & 47.979 & 61.979 & 71.050 & 56.621 \\ 0.0156 & 106.93 & 69.132 & -73.363 & -116.88 \\ 0.2137 & 0.0706 & 0.2005 & -0.0056 & -0.2459 \\ 0.0074 & 0.4595 & 0.0311 & -0.0398 & -0.5191 \\ -0.2163 & 0.0157 & 0.4482 & 0.4931 & 0.0137 \end{bmatrix}$$

The identified open-loop Kalman filter gain K in the discrete-time domain from test data is

$$\begin{bmatrix} 0.2406 & -0.0343 & -0.4772 & -3.0053 & 4.3355 \\ -3.3760 & 0.7401 & 4.5882 & -0.3932 & 0.5876 \\ -6.2835 & -0.6517 & -5.7352 & 0.3747 & 0.0586 \\ 0.2171 & -0.1556 & 0.4695 & 6.9133 & 3.8937 \\ 0.4699 & 6.4395 & -2.2154 & 0.0252 & 0.5621 \\ 4.9020 & -0.8834 & 5.2593 & -0.0552 & -0.3753 \\ -0.2149 & 0.2539 & -0.2921 & 0.8056 & -4.2704 \\ -3.1995 & -1.1073 & 2.8743 & 0.2870 & 0.2235 \\ 0.2149 & 0.0016 & -0.0996 & 5.5887 & 2.6125 \\ 0.4253 & -4.5657 & -1.3187 & 0.2038 & -0.2893 \end{bmatrix}$$

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